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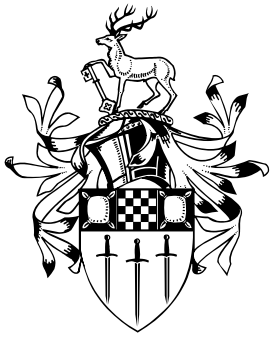

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INDETERMINACY WITH INFLATION-FORECAST-BASED RULES IN A TWO-BLOC MODEL

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Indeterminacy with Inflation-Forecast-Based Rules in a Two-Bloc Model*

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Abstract

We examine the performance of forward-looking inflation-forecast-based rules in open economies. In a New Keynesian two-bloc model, a methodology first employed by Batini and Pearlman (2002) is used to obtain analytically the feedback parameters/horizon pairs associated with unique and stable equilibria. Three key findings emerge: first, indeterminacy occurs for any value of the feedback parameter on inflation if the forecast horizon lies too far into the future. Second, the problem of indeterminacy is intrinsically more serious in the open economy. Third, the problem is compounded further in the open economy when central banks respond to expected consumer, rather than producer price inflation.

JEL Classification: E52, E37, E58

Keywords: Taylor rules, inflation-forecast-based rules, indeterminacy, open economy

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1 Introduction

Under inflation targeting, the task of the central bank is to alter monetary conditions to keep inflation close to a pre-announced target. One class of rules widely proposed under inflation targeting are 'inflation-forecast-based' (IFB) rules (Batini and Haldane (1999)). IFB rules are 'simple' rules as in Taylor (1993), but where the policy instrument responds to deviations of expected, rather than current inflation from target. The horizon in the rule is a policy parameter, alongside the feedback parameters. In most applications, the inflation forecasts underlying IFB rules are taken to be the endogenous rational-expectations forecasts conditional on an intertemporal equilibrium of the model. These rules are of specific interest because similar reaction functions are used in the Quarterly Projection Model of the Bank of Canada (see Coletti *et al.* (1996)), and in the Forecasting and Policy System of the Reserve Bank of New Zealand (see Black *et al.* (1997)) – two prominent inflation targeting central banks. As shown in Clarida *et al.* (2000) – CGG (2000) henceforth– and Castelnuovo (2003), estimates of IFB-type rules appear to be a good fit to the actual monetary policy in the US and Europe of recent years.

However, IFB rules have been criticized on various grounds. Svensson (2001, 2003) criticizes Taylor-type rules in general and argues for policy based on explicit maximization procedures.[1] Much of the literature, however, focuses on a more specific possible problem with Taylor-type rules – that of equilibrium indeterminacy when they are forward-looking. *Nominal* indeterminacy arising from an interest rate rule was first shown by Sargent and Wallace (1975) in a flexible price model. In sticky-price New Keynesian models this nominal indeterminacy disappears because the previous period's price level serves as a nominal anchor. But now a problem of *real indeterminacy* emerges with IFB rules taking two forms: if the response of interest rates to a rise in expected inflation is insufficient, then real interest rates fall thus raising demand and confirming any exogenous expected inflation (see CGG (2000) and Batini and Pearlman (2002)). But indeterminacy is also possible if the rule is overly aggressive (Bernanke and Woodford (1997); Batini and Pearlman (2002); Giannoni and Woodford (2002)).[2] Here we extend this literature by studying the

[1] We discuss his critique in a longer working paper version of this paper, Batini *et al.* (2004), BLP henceforth.

[2] Both types of real indeterminacy can be illustrated in a very simple closed economy model: consider a special case of 'Phillips Curve' set out in this paper, $\pi_t = \mathcal{E}_t(\pi_{t+1}) + ay_t$, where π_t denotes inflation

uniqueness and stability conditions for an equilibrium under IFB rules for various feedback horizons in open economies, paying particular attention to possible implications for the US/euro area region.

This paper employs the same root locus methodology employed by Batini and Pearlman (2002) in the closed-economy context to identify analytically the feedback parameters/horizon pairs that are associated with unique and stable equilibria in a New Keynesian sticky-price two-bloc model similar to Benigno and Benigno (2001) – BB henceforth– and Clarida *et al.* (2002) – CGG (2002) henceforth. We modify the BB/CGG (2002) model to include habit formation in consumption and inflation indexing, changes that help to improve the ability of the model to capture the inflation and output dynamics observed in the euro area and the US. We also generalize the model to allow for the possibility that agents in the two blocs exhibit home bias in consumption patterns. This produces short-run and long-run deviations from consumption-based purchasing power parity, and improves the model's ability to replicate the large and protracted swings in the real euro/dollar rate observed since the launch of the euro.

Analyzing a two-bloc model is particularly interesting because it allows us to explore the implications for rational-expectations equilibria of concurrent monetary policy strategies of the European Central Bank (ECB) and the Federal Reserve. In addition, by assuming that the two blocs are identical in both fundamental parameters and in policy, we can use the Aoki (1981) decomposition of the model into sum and differences forms; we can then examine whether findings in the literature on the stability and uniqueness of equilibria based on a closed-economy assumption translate to the open-economy case.

Three key findings emerge from this paper. First, we find that indeterminacy occurs for any value of the feedback parameter on inflation in the forward-looking rule if the forecast

and y_t is the deviation of output from its equilibrium level. Close the model with an ad hoc 'IS' curve $y_t = -b(i_t - \mathcal{E}_t(\pi_{t+1}))$ where i_t is the nominal interest rate which is set according to an IFB-Taylor rule $i_t = \theta\mathcal{E}_t(\pi_{t+1}) + \mu y_t$. Substituting out for y_t and i_t we arrive at $\mathcal{E}_t(\pi_{t+1}) = \frac{1+b\mu}{1+b\mu-ab(\theta-1)}\pi_t$ which has a unique rational expectations solution $\pi_t = 0$ iff $\frac{1+b\mu}{1+b\mu-ab(\theta-1)} > 1$ and a stable trajectory, tending to zero inflation in the long run, consistent with any initial inflation rate otherwise– that is there is indeterminacy if $\theta < 1$ or $\theta > 1 + \frac{2(1+b\mu)}{ab}$. In the latter case, overly aggressive feedback produces cycles of positive and negative inflation. Thus the inclusion of a feedback on output reduces the region of indeterminacy. Empirical estimates of μ appear to be small, as discussed in section 2. So, in our subsequent analysis, we focus exclusively on 'pure' IFB rules, i.e. rules without an output gap term.

horizon lies too far into the future.[3] This reaffirms, for the open-economy case, results found in the literature for the closed-economy case. Second, we find that the problem of indeterminacy is intrinsically more serious in an open than in a closed economy. Third, we find the problem is compounded further in the open economy when central banks in the two blocs respond to expected consumer, rather than expected producer price, inflation.

The plan of the paper is as follows. Section 2 offers an overview of the main related papers. Section 3 sets out our two-bloc model. Section 4 uses the root locus analysis technique to investigate the stability and uniqueness conditions for IFB rules based on producer price or consumer price inflation, allowing for the possibility of home consumption bias. Section 5 offers some concluding remarks.

2 Recent Related Literature

Perhaps the best-known theoretical result in the literature on IFB rules is that to avoid indeterminacy the monetary authority must respond aggressively, that is with a coefficient above unity, but not excessively large, to expected inflation in the closed-economy context (see, among others, CGG (2000) and, in the small-open-economy context, see De Fiore and Liu (2002)and Zanna (2003)). Bullard and Mitra (2001) reworked this result in a closed-economy model where private agents form forecasts using recursive learning algorithms. Empirically, both the Federal Reserve in the post-Volker era and European monetary authorities post 1980 appear to have indeed responded to expected inflation with a cofficient greater than 1 (see CGG (2000); Castelnuovo (2003); Faust *et al.* (2001)).[4]

[3] The fact that forward-looking behavior is a source of indeterminacy can again be illustrated using the simple model of the previous footnote. Consider a rule involving a feedback on *current* inflation and the current output gap: $i_t = \theta\pi_t + \mu y_t$. Then re-working the analysis we arrive at $E_t(\pi_{t+1}) = \frac{1+b\mu+a\theta(1+(b+1)\mu)}{1+b\mu+a(1+(b+1)\mu)}$ which has a unique RE solution π_t iff $\theta > 1$. For this current-looking rule there is no upper-bound on θ: all values above 1 ensure determinacy.

[4] Although empirical evidence seems to lend support to the idea that the US and European central banks follow IFB-type rules, the Lucas Critique suggests that there is a logical distinction between observing that a simple reduced-form relationship holds between variables and assuming that such a relation holds as a structural equation. For example, Tetlow (2000) demonstrates that a Taylor rule may seem to explain US monetary policy even if monetary policy is set optimally, conditioning on literally hundreds of state variables.

The case for an aggressive rule however has been questioned by a number of recent theoretical studies. First, the result depends entirely on: (a) the way in which money is assumed to enter preferences and technology; and (b) how flexible prices are. In the closed-economy context, both Carlstrom and Fuerst (2000) and Benhabib *et al.* (2001) showed, for example, that with sticky prices the result is overturned when money enters the utility function either as in Sidrauski-Brock or via more realistic cash-in-advance timing assumptions.[5] With these assumptions, if the monetary authority responds aggressively to future expected inflation it makes indeterminacy more likely, whereas if it does so to past inflation it makes determinacy less likely.

Second, the result rests on the assumption that, in its attempt to look forward, the central bank responds only to next quarter's inflation forecast, not to forecasts at later quarters. However, real-world procedures typically involve stabilizing inflation in the medium-run, one to two years out. It follows that the above result may not translate into sound policy prescriptions for inflation targeters. Complementing numerical results by Levin *et al.* (2001)–LWW henceforth– Batini and Pearlman (2002) showed analytically that IFB rules may lead to indeterminacy in the standard IS-AS optimizing forward-looking model used, for example, by Woodford (1999). Below we build on this work to study indeterminacy with IFB rules responding beyond one quarter in the context of a dynamic two-bloc New-Keynesian model. In doing so we consider the impact of various degrees of openness and price flexibility on our indeterminacy results, but stick to the conventional timing used in most open-economy optimizing-agents models whereby real money entering the utility function refers to end-of-period balances.

3 The Model

Our model is essentially a generalization of CGG (2002) and BB to incorporate a bias for consumption of home-produced goods, habit formation in consumption, and Calvo price setting with indexing of prices for those firms who, in a particular period, do not re-optimize their prices. The latter two aspects of the model follow Christiano *et al.* (2001)

[5] De Fiore and Liu (2002) assume this latter type of cash-in-advance assumption and show, in the context of a small open-economy model, that indeterminacy results are sensitive to the various assumptions on the timing of transactions.

and, as with these authors, our motivation is an empirical one: to generate sufficient inertia in the model so as to enable it, in calibrated form, to reproduce commonly observed output, inflation and nominal interest rate responses to exogenous shocks.

There are two equally-sized[6] symmetric blocs with the same household preferences and technologies. In each bloc there is one traded risk-free nominal bond denominated in the home bloc's currency. The exchange rate is perfectly flexible. A final homogeneous good is produced competitively in each bloc using a CES technology consisting of a continuum of differentiated non-traded goods. Intermediate goods producers and household suppliers of labor have monopolistic power. Nominal prices of intermediate goods, expressed in the currency of producers, are sticky.

The monetary policy of the central banks in the two blocs takes the same form; namely, that of an IFB nominal interest rate rule with identical parameters. The money supply accommodates the demand for money given the setting of the nominal interest rate according to such a rule. Since the paper is exclusively concerned with the possible indeterminacy or instability of IFB rules, we confine ourselves to a *perfect foresight equilibrium* in a deterministic environment with monetary policy responding to unanticipated transient exogenous TFP shocks. The decisions of households and firms are as follows:

3.1 Households

A representative household r in the 'home' bloc maximizes

$$\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t(r) - H_t)^{1-\sigma}}{1-\sigma} + \chi \frac{\left(\frac{M_t(r)}{P_t}\right)^{1-\varphi}}{1-\varphi} - \kappa \frac{N_t(r)^{1+\phi}}{1+\phi} \right] \tag{1}$$

where $\mathcal{E}_t$ is the expectations operator indicating expectations formed at time t, $C_t(r)$ is an index of consumption, $N_t(r)$ are hours worked, H_t represents the habit, or desire not to differ too much from other consumers, and we choose it as $H_t = hC_{t-1}$, where C_t is the average consumption index and $h \in [0, 1)$. When $h = 0$, $\sigma > 1$ is the risk aversion parameter (or the inverse of the intertemporal elasticity of substitution)[7]. $M_t(r)$ are

[6]The population in each bloc is normalized at unity. It is straightforward to allow for different sized blocs, as in CGG (2002) and BB. Then in the Aoki decomposition, aggregates must be population-weighted and differences expressed in per capita terms.

[7]When $h \neq 0$, σ is merely an index of the curvature of the utility function.

end-of-period nominal money balances. An analogous symmetric intertemporal utility is defined for the 'foreign' representative household and the corresponding variables (such as consumption) are denoted by $C_t^*(r)$, etc.

The representative household r must obey a budget constraint:

$$P_t C_t(r) + D_t(r) + M_t(r) = W_t(r)N_t(r) + (1 + i_{t-1})D_{t-1}(r) + M_{t-1}(r) + \Gamma_t(r) \qquad (2)$$

where P_t is a price index, $D_t(r)$ are end-of-period holdings of riskless nominal bonds with nominal interest rate i_t over the interval $[t, t+1]$. $W_t(r)$ is the wage and $\Gamma_t(r)$ are dividends from ownership of firms. In addition, if we assume that households' labour supply is differentiated with elasticity of supply η, then (as we shall see below) the demand for each consumer's labor is given by

$$N_t(r) = \left(\frac{W_t(r)}{W_t}\right)^{-\eta} N_t \qquad (3)$$

where $W_t = \left[\int_0^1 W_t(r)^{1-\eta} dr\right]^{\frac{1}{1-\eta}}$ is an average wage index and $N_t = \int_0^1 N_t(r)dr$ is aggregate employment.

We assume that the consumption index depends on the consumption of a single type of final good in each of two identically sized blocs, and is given by

$$C_t(r) = C_{Ht}(r)^{1-\omega} C_{Ft}(r)^{\omega} \qquad (4)$$

where $\omega \in [0, \frac{1}{2}]$ is a parameter that captures the degree of 'openness'. If $\omega = 0$ we have autarky, while the other extreme of $\omega = \frac{1}{2}$ gives us the case of perfect integration. For $\omega < \frac{1}{2}$ there is some degree of 'home bias'.[8] If P_{Ht}, P_{Ft} are the domestic prices of the two types of good, then the optimal intra-temporal decisions are given by standard results:

$$P_{Ht}C_{Ht}(r) \;\; = \;\; (1-\omega)P_t C_t(r)\,; \;\; P_{Ft}C_{Ft}(r) = \omega P_t C_t(r) \qquad (5)$$

with the consumer price index P_t given by

$$P_t = k P_{Ht}^{1-\omega} P_{Ft}^{\omega} \qquad (6)$$

where $k = (1-\omega)^{-(1-\omega)}\omega^{-\omega}$. Assume that the law of one price holds i.e. prices in home and foreign blocs are linked by $P_{Ht} = S_t P_{Ht}^*, P_{Ft} = S_t P_{Ft}^*$ where P_{Ht}^* and P_{Ft}^* are the

[8]The effect of home bias in open economies is also studied in Corsetti *et al.* (2002) and De Fiore and Liu (2002).

foreign currency prices of the home and foreign-produced goods and S_t is the nominal exchange rate. Let $P_t^* = k{P_{Ht}^*}^{\omega}{P_{Ft}^*}^{1-\omega}$ be the foreign consumer price index corresponding to (6). Then it follows that the real exchange rate $E_t = \frac{S_t P_t^*}{P_t}$ and the terms of trade $\mathcal{T} = \frac{P_{Ht}}{P_{Ft}}$ are related by the relationship

$$E_t \equiv \frac{S_t P_t^*}{P_t} = \mathcal{T}^{2\omega-1} \tag{7}$$

Thus (since $2\omega - 1 \leq 0$), as the real exchange rate appreciates (i.e., E_t falls) the terms of trade improve, except at the extreme of perfect integration where $\omega = \frac{1}{2}$. Then $E_t = 1$ and the law of one price applies to the aggregate price indices.

In a perfect foresight equilibrium, maximizing (1) subject to (2) and (3) and imposing symmetry on households (so that $C_t(r) = C_t$, etc) yields standard results:

$$1 = \beta(1+i_t)\left(\frac{C_{t+1}-H_{t+1}}{C_t - H_t}\right)^{-\sigma}\frac{P_t}{P_{t+1}} \tag{8}$$

$$\left(\frac{M_t}{P_t}\right)^{-\varphi} = \frac{(C_t-H_t)^{-\sigma}}{\chi P_t}\left[\frac{i_t}{1+i_t}\right] \tag{9}$$

$$\frac{W_t}{P_t} = \frac{\kappa}{(1-\frac{1}{\eta})}N_t^{\phi}(C_t - H_t)^{\sigma} \tag{10}$$

(8) is the familiar Keynes-Ramsey rule adapted to take into account of the consumption habit. In (9), the demand for money balances depends positively on consumption relative to habit and negatively on the nominal interest rate. Given the central bank's setting of the latter, (9) is completely recursive to the rest of the system describing our macro-model and will be ignored in the rest of the paper. (10) reflects the market power of households arising from their monopolistic supply of a differentiated factor input with elasticity η.

Households can accumulate assets in the form of either home or foreign bonds. Uncovered interest rate parity then gives

$$1 + i_t = \frac{S_{t+1}}{S_t}(1 + i_t^*) \tag{11}$$

where i_t^* is the interest rate paid on nominal bonds denominated in foreign currency.

3.2 Firms

Competitive final goods firms use a continuum of non-traded intermediate goods according to a constant returns CES technology to produce aggregate output

$$Y_t = \left(\int_0^1 Y_t(m)^{(\zeta-1)/\zeta}dm\right)^{\zeta/(\zeta-1)} \tag{12}$$

where ζ is the elasticity of substitution. This implies a set of demand equations for each intermediate good m with price $P_{Ht}(m)$ of the form

$$Y_t(m) = \left(\frac{P_{Ht}(m)}{P_{Ht}}\right)^{-\zeta} Y_t \tag{13}$$

where $P_{Ht} = \left[\int_0^1 P_{Ht}(m)^{1-\zeta} dm\right]^{\frac{1}{1-\zeta}}$. P_{Ht} is an aggregate intermediate price index, but since final goods firms are competitive and the only inputs are intermediate goods, it is also the domestic price level.

In the intermediate goods sector each good m is produced by a single firm m using only differentiated labour with another constant returns CES technology:

$$Y_t(m) = A_t \left(\int_0^1 N_{tm}(r)^{(\eta-1)/\eta} dr\right)^{\eta/(\eta-1)} \tag{14}$$

where $N_{tm}(r)$ is the labour input of type r by firm m and A_t is an exogenous shock capturing shifts to trend total factor productivity (TFP) in this sector. Minimizing costs $\int_0^1 W_t(r) N_{tm}(r) dr$ and aggregating over firms leads to the demand for labor as shown in (3). In a equilibrium of equal households and firms, all wages adjust to the same level W_t and it follows that $Y_t = A_t N_t$.

For later analysis it is useful to define the real marginal cost as the wage relative to domestic producer price. Using (10) and $Y_t = A_t N_t$ this can be written as

$$MC_t \equiv \frac{W_t}{A_t P_{Ht}} \quad = \quad \frac{\kappa}{(1-\frac{1}{\eta})A_t} \left(\frac{Y_t}{A_t}\right)^{\phi} (C_t - H_t)^{\sigma} \left(\frac{P_{Ft}}{P_{Ht}}\right)^{\omega} \tag{15}$$

Now we assume that there is a probability of $1-\xi$ at each period that the price of each intermediate good m is set optimally to $P^O_{Ht}(m)$. If the price is not re-optimized, then it is indexed to last period's aggregate producer price inflation.[9] With indexation parameter $\gamma \geq 0$, this implies that successive prices with no reoptimization are given by $P^O_{Ht}(m)$, $P^O_{Ht}(m)\left(\frac{P_{Ht}}{P_{H,t-1}}\right)^{\gamma}$, $P^O_{Ht}(m)\left(\frac{P_{H,t+1}}{P_{H,t-1}}\right)^{\gamma}$, ... For each intermediate producer m the objective is at time t to choose $P^O_{Ht}(m)$ to maximize discounted profits

$$\mathcal{E}_t \sum_{k=0}^{\infty} \left(\frac{\xi}{1+i_t}\right)^k Y_{t+k}(m) \left[P^O_{Ht}(m) \left(\frac{P_{H,t+k-1}}{P_{H,t-1}}\right)^{\gamma} - \frac{W_{t+k}}{A_t}\right] \tag{16}$$

[9] Thus we can interpret $\frac{1}{1-\xi}$ as the average duration for which prices are left unchanged.

given i_t (since firms are atomistic), subject to (13). The solution to this is

$$\mathcal{E}_t \sum_{k=0}^{\infty} \left(\frac{\xi}{1+i_t}\right)^k Y_{t+k}(m) \left[P_{Ht}^O(m) \left(\frac{P_{H,t+k-1}}{P_{H,t-1}}\right)^{\gamma} - \frac{1}{(1-1/\zeta)} \frac{W_{t+k}}{A_t}\right] = 0 \qquad (17)$$

and by the law of large numbers the evolution of the price index is given by

$$P_{H,t+1}^{1-\zeta} = \xi \left(P_{Ht} \left(\frac{P_{Ht}}{P_{H,t-1}}\right)^{\gamma}\right)^{1-\zeta} + (1-\xi)(P_{H,t+1}^O)^{1-\zeta} \qquad (18)$$

3.3 The Equilibrium and the Trade Balance

In equilibrium, goods markets, money markets and the bond market all clear. Equating the supply and demand of the home consumer good and using (5) and its foreign counterpart we obtain

$$Y_t = AN_t = C_{Ht} + C_{Ht}^* = \frac{P_t}{P_{Ht}} \left[(1-\omega)C_t + \omega E_t C_t^*\right] \qquad (19)$$

Given interest rates i, i^* (expressed later in terms of a IFB rule) the money supply is fixed by the central banks to accommodate money demand. By Walras' Law we can dispense with the bond market equilibrium condition. Then a *perfect foresight equilibrium* is defined at $t = 0$ as sequences C_t, D_t, C_{Ht}, C_{Ft}, P_{Ht}, P_{Ft}, P_t, M_t, W_t, Y_t, N_t, P_{Ht}^0, 12 foreign counterparts C_t^*, D_t^*, etc, E_t, and S_t, given past price indices and exogenous TFP processes.

Combining the Keynes-Ramsey equations with the UIP condition we have that

$$\frac{P_t^*}{P_t} \left(\frac{C_t - hC_{t-1}}{C_t^* - hC_{t-1}^*}\right)^{-\sigma} = \frac{S_{t+1}}{S_t} \frac{P_{t+1}^*}{P_{t+1}} \left(\frac{C_{t+1} - hC_t}{C_{t+1}^* - hC_t^*}\right)^{-\sigma} \qquad (20)$$

Let $z_t = \frac{S_t P_t^*}{P_t} \left(\frac{C_t - hC_{t-1}}{C_t^* - hC_{t-1}^*}\right)^{-\sigma}$. Then (20) implies that $z_{t+1} = z_t$. We consider a linearization in the vicinity of a symmetric steady state, $\bar{z} = 1$. From the transient nature of the shocks it follows that this steady state remains unchanged and hence $z_t = 1$ in any stable rational expectations equilibrium. Therefore[10]

$$\left(\frac{C_t - hC_{t-1}}{C_t^* - hC_{t-1}^*}\right)^{-\sigma} = \frac{P_t}{S_t P_t^*} = \frac{1}{E_t} \qquad (21)$$

[10]In a stochastic setting with complete asset markets, (21) is simply the risk-sharing condition for consumption, because it equates marginal rate of substitution to relative price, as would be obtained if utility were being jointly maximized by a social planner (see Sutherland (2002)).

The model as it stands with habit persistence ($h > 0$), $\sigma > 1$ and $\omega \in [0, \frac{1}{2})$ exhibits net foreign asset dynamics. This can be shown by writing the trade balance TB_t in the home bloc as exports minus imports denominated its own currency:

$$\text{TB}_t = P_{Ht}C^*_{Ht} - P_{Ft}C_{Ft} = \omega\left(\frac{P_{Ht}}{P^*_{Ht}}P^*_t C^*_t - P_t C_t\right) = \omega P_t(E_t C^*_t - C_t) \tag{22}$$

using (5), the law of one price $P_{Ht} = S_t P^*_{Ht}$, and recalling the definition $E_t \equiv \frac{S_t P^*_t}{P_t}$. Therefore there are net foreign asset dynamics unless $C_t = E_t C^*_t$. This is only compatible with (21) if either $\omega = \frac{1}{2}$ (no home bias), in which case $E_t = 1$, and we start off with balanced trade; or if $\sigma = 1$ and $h = 0$ (no habit persistence).

3.4 Linearization

We linearize around a baseline symmetric steady state in which consumption and prices in the two blocs are equal and constant. Then inflation is zero, $E_t = \bar{E} = 1$ and hence from (22) trade is balanced. Output is then at its sticky-price, imperfectly competitive natural rate and from the Keynes-Ramsey condition (8) the nominal rate of interest is given by $\bar{\imath} = \frac{1}{\beta} - 1$. Now define all lower case variables as proportional deviations from this baseline steady state.[11] Home producer and consumer inflation are defined as $\pi_{Ht} \equiv \frac{P_{Ht} - P_{H,t-1}}{P_{H,t-1}} \simeq p_{Ht} - p_{H,t-1}$ and $\pi_t \equiv \frac{P_t - P_{t-1}}{P_{t-1}} \simeq p_t - p_{t-1}$ respectively. Similarly, define foreign producer inflation and consumer price inflation. Combining (17) and (18), we can eliminate P^0_{Ht} to obtain in linearized form

$$\pi_{Ht} = \frac{\beta}{1+\beta\gamma}\mathcal{E}_t\pi_{H,t+1} + \frac{\gamma}{1+\beta\gamma}\pi_{H,t-1} + \frac{(1-\beta\xi)(1-\xi)}{(1+\beta\gamma)\xi}mc_t \tag{23}$$

The linearized version of the real marginal cost for producers of intermediate goods in the home bloc, (15), is given by

$$mc_t = -(1+\phi)a_t + \frac{\sigma}{1-h}(c_t - hc_{t-1}) + \phi y_t + \omega(s_t + p^*_{Ft} - p_{Ht}) \tag{24}$$

The first term on the right-hand-side of (24) is a TFP shock. The second term is *a risk-sharing effect*: a rise in habit-adjusted consumption leads to an increase in the real wage

[11] That is, for a typical variable X_t, $x_t = \frac{X_t - \bar{X}}{\bar{X}} \simeq \log\left(\frac{X_t}{\bar{X}}\right)$ where $\bar{X}$ is the baseline steady state. The interest rate however is now expressed as an absolute deviation about $\bar{\imath}$.

(see (10)) and hence the marginal cost. The last term is a *terms of trade effect,* which implies that marginal costs falls if the terms of trade, $p_{Ht} - s_t - p^*_{Ft}$ in linearized form, rises.

Linearizing the remaining equations (7), (8), (11), (19) and (21) yields

$$\begin{aligned}
\pi_t - \pi^*_t &= 2\omega(s_t - s_{t-1}) + (1-2\omega)(\pi_{Ht} - \pi^*_{Ft}) && (25)\\
c_t - \frac{h}{1+h}c_{t-1} &= \frac{1}{1+h}\mathcal{E}_t c_{t+1} - \frac{1-h}{(1+h)\sigma}(i_t - \mathcal{E}_t\pi_{t+1}) && (26)\\
\mathcal{E}_t\Delta s_{t+1} &= i_t - i^*_t && (27)\\
y_t &= (1-\omega)c_t + \omega c^*_t - 2\omega(1-\omega)(p_{Ht} - s_t - p^*_{Ft}) && (28)\\
\sigma(c^*_t - c_t - h(c^*_{t-1} - c_{t-1})) &= -e_t = (1-2\omega)(p_{Ht} - s_t - p^*_{Ft}) && (29)
\end{aligned}$$

Note that (28) and its foreign counterpart imply that $y_t + y^*_t = c_t + c^*_t$. Also note that for the case when there is no home bias, $\omega = 1/2$, then (25) reduces to relative purchasing power parity for consumer price inflation.

Turning to *spillover effects* in our linearized form of the model, let us focus on the case of no home bias, $\omega = 1/2$. Then from (28) and (24) we obtain

$$mc_t = -(1+\phi)a_t + \frac{\sigma}{2(1-h)}\left[y_t - hy_{t-1} + y^*_t - hy^*_{t-1}\right] + \phi y_t + \frac{1}{2}\left[y_t - y^*_t\right] \qquad (30)$$

It follows that the elasticity of marginal cost for intermediate goods home producers with respect to domestic and foreign current output, given output at time $t-1$, are given by $\kappa_0 \equiv \frac{\partial mc_t}{\partial y_t}$ and $\kappa_1 \equiv \frac{\partial mc_t}{\partial y^*_t}$ where

$$\kappa_0 = \frac{\sigma}{2(1-h)} + \frac{1}{2} + \phi\,; \qquad \kappa_1 = \frac{\sigma}{2(1-h)} - \frac{1}{2} \qquad (31)$$

(31) indicates that the risk-sharing effect exceeds the terms of trade effect and there is positive spillover from output onto the marginal cost of the second bloc (implying a negative spillover on output) iff $\frac{\sigma}{1-h} > 1$ in the short-run (i.e., given output in period $t-1$).[12] Iff $\frac{\sigma}{1-h} = 1$, the risk-sharing and terms of trade effect cancel and there are no spillover effects. Empirical estimates discussed in Appendix C of BLP suggest that $\sigma > 1$, so under this calibration spillover effects on output are negative. The effect of introducing habit is to enhance the risk-sharing effect and thus increase these negative short-run spillovers.

[12] If $h = 0$, this replicates the result in CGG (2002).

3.5 Sum and Difference Systems

Since the economies are symmetric, the easiest way of analyzing them is to use the sum and difference systems, as introduced by Aoki (1981). We denote all sums of home and foreign variables with the superscript S, while we denote differences by D. The first thing to note when inspecting the equations above is that the sum system is independent of home bias, and can be written as

$$\begin{aligned}
\pi_t^S &= \frac{\beta}{1+\beta\gamma}\mathcal{E}_t\pi_{t+1}^S + \frac{\gamma}{1+\beta\gamma}\pi_{t-1}^S \\
&+ \frac{(1-\beta\xi)(1-\xi)}{(1+\beta\gamma)\xi}[(\phi+\frac{\sigma}{1-h})y_t^S - \frac{\sigma h}{1-h}y_{t-1}^S - (1+\phi)a_t^S] \qquad (32)\\
y_t^S &= \frac{h}{1+h}y_{t-1}^S + \frac{1}{1+h}\mathcal{E}_t y_{t+1}^S - \frac{1-h}{(1+h)\sigma}(i_t^S - \mathcal{E}_t\pi_{t+1}^S) \qquad (33)
\end{aligned}$$

where $\pi^S = \pi_H + \pi_F^*$, $y^S = y + y^*$, and we note that $\pi_H + \pi_F^* = \pi + \pi^*$.

However the difference system does depend on the home bias parameter, ω, Writing $\pi^D = \pi_H - \pi_F^*$, $y^D = y - y^*$, etc., it can be written as

$$\begin{aligned}
\pi_t^D &= \frac{\beta}{1+\beta\gamma}\mathcal{E}_t\pi_{t+1}^D + \frac{\gamma}{1+\beta\gamma}\pi_{t-1}^D + \frac{(1-\beta\xi)(1-\xi)}{(1+\beta\gamma)\xi}mc_t^D \qquad (34)\\
mc_t^D &= -(1+\phi)a_t^D + \frac{\sigma}{1-h}(c_t^D - hc_{t-1}^D) + \phi y_t^D + 2\omega(s_t + p_{Ft}^* - p_{Ht}) \qquad (35)\\
c_t^D &= hc_{t-1}^D + \frac{(2\omega-1)}{\sigma}(p_{Ht} - s_t - p_{Ft}^*) \qquad (36)\\
y_t^D &= (1-2\omega)c_t^D - 4\omega(1-\omega)(p_{Ht} - s_t - p_{Ft}^*) \qquad (37)\\
\mathcal{E}_t\Delta s_{t+1} &= i_t^D \qquad (38)
\end{aligned}$$

For the case of no home consumption bias ($\omega = \frac{1}{2}$), taking first differences of (37) and using (38) we have

$$\mathcal{E}_t y_{t+1}^D - y_t^D = i_t^D - \mathcal{E}_t\pi_{t+1}^D \qquad (39)$$

In addition, when there is no home bias, the remainder of the difference system reduces to

$$\pi_t^D = \frac{\beta}{1+\beta\gamma}\mathcal{E}_t\pi_{t+1}^D + \frac{\gamma}{1+\beta\gamma}\pi_{t-1}^D + \frac{(1-\beta\xi)(1-\xi)}{(1+\beta\gamma)\xi}(1+\phi)(y_t^D - a_t^D) \qquad (40)$$

Note, as with other models of the same New Keynesian genre, there is a small *long-run* inflation-unemployment trade-off.

The sum and difference systems can now be set up in state-space form given the nominal interest rate rule. This Aoki decomposition enables us to decompose the open economy

into two decoupled dynamic systems; the sum system, that captures the properties of a closed world economy, and a difference system that instead portrays the contribution of openness. In principle, we could close the model with a number of different Taylor-type rules but here we choose to focus on IFB rules that feedback exclusively on expected inflation. As discussed in BLP, it is possible to design *optimal* IFB rules within the constraints defined by the rule. However, the literature on determinacy, to which our paper contributes, has a more modest objective of providing guidelines to policymakers in the form of simple criteria for avoiding very bad outcomes that lead to multiple equilibria or explosive behaviour. In our set-up, these guidelines focus on the choice of feedback, interest rate smoothing and feedback horizon parameters. We now pursue this objective by looking at how such guidelines are affected when we proceed from the closed to the open economy, and by the degree of openness in the latter.

4 The Stability and Determinacy of IFB Rules

This section studies two particular forms of simple rule, IFB rules either of the form

$$i_t = \rho i_{t-1} + \theta(1-\rho)\mathcal{E}_t\pi_{t+j} \tag{41}$$

where $j \geq 0$ is the forecast horizon, which is a feedback on consumer price inflation, or of the form

$$i_t = \rho i_{t-1} + \theta(1-\rho)\mathcal{E}_t\pi_{Ht+j} \tag{42}$$

which is a feedback on producer price inflation.[13] We assume that the foreign bloc has a similar rule with the same parameters and forecast horizon.

With rules (41) or (42), policymakers set the nominal interest rate so as to respond to deviations of the inflation term from target. In addition, policymakers smooth rates, in line with the idea that central banks adjust the short-term nominal interest rate only partially towards the long-run inflation target, which is set to zero for simplicity in our set-up.[14] The parameter $\rho \in [0,1)$ measures the degree of interest rate smoothing. j

[13]Both rules are in absolute deviation form about the baseline steady state and could represent the feedback component of monetary policy that complements a (possibly optimal) open-loop trajectory.

[14]For instance (41) can be written as $\Delta i_t = \frac{1-\rho}{\rho}[\theta\mathcal{E}_t\pi_{t+j} - i_t]$ which is a partial adjustment to a static IFB rule $i_t = \theta\mathcal{E}_t\pi_{t+j}$.

is the feedback horizon of the central bank. When $j = 0$, the central bank feeds back from current dated variables only. When $j > 0$, the central bank feeds back instead from deviations of forecasts of variables from target. Finally, $\theta > 0$ is the feedback parameter: the larger is θ, the faster is the pace at which the central bank acts to eliminate the gap between expected inflation and its target value. We now show that, for given degrees of interest rate smoothing ρ, the stabilizing characteristics of these rules depend both on the magnitude of θ and the length of the feedback horizon j.

4.1 Conditions for Uniqueness and Stability

To understand better how the precise combination of the pair (j, θ), IFB rules can lead the economy into instability or indeterminacy consider the sum form of the model economy (32) and (33) with interest rate rules of the form (41) with $j = 0, 1$. Shocks to TFP are exogenous stable processes and play no part in the stability analysis. We therefore set $a_t^S = 0$. Write the sum economy and the rule in state space form as

$$\begin{bmatrix} \mathsf{z}_{t+1} \\ \mathcal{E}_t \mathsf{x}_{t+1} \end{bmatrix} = A \begin{bmatrix} \mathsf{z}_t \\ \mathsf{x}_t \end{bmatrix} + B i^S \quad \text{where} \quad i^S = D \begin{bmatrix} \mathsf{z}_t \\ \mathsf{x}_t \end{bmatrix} \tag{43}$$

where $\mathsf{z}_t = [y_{t-1}^S, \pi_{t-1}^S, i_{t-1}^S,]$ is a vector of predetermined variables and $\mathsf{x}_t = [y_t^S, \pi_t^S]$ are non-predetermined variables. This gives the system under control as

$$\begin{bmatrix} \mathsf{z}_{t+1} \\ \mathcal{E}_t \mathsf{x}_{t+1} \end{bmatrix} = [A + BD] \begin{bmatrix} \mathsf{z}_t \\ \mathsf{x}_t \end{bmatrix} \tag{44}$$

The condition for a stable and unique equilibrium depends on the magnitude of the eigenvalues of the matrix $A + BD$. If the number of eigenvalues outside the unit circle is equal to the number of non-predetermined variables, the system has a unique stable equilibrium with saddle-path $x_t = -Nz_t$ where $N = N(D)$. (See Blanchard and Kahn (1980); Currie and Levine (1993)). In our sum model under control, with $j = 0, 1$, there are 3 non-predetermined variables in z_t and 2 non-predetermined variables in x_t. Instability occurs when the number of eigenvalues of $A + BD$ outside the unit circle is larger than the number of non-predetermined variables. By contrast, indeterminacy occurs when the number of eigenvalues of $A + BD$ outside the unit circle is smaller than the number of non-predetermined variables. This implies that when a shock displaces the economy from

its steady state, there are an infinite number of possible paths leading back to equilibrium. With forward-looking rules this can happen when policymakers respond to private sector's inflation expectations and these in turn are driven by non-fundamental exogenous random shocks (i.e. not based on preferences or technology), usually referred to as 'sunspots'. If policymakers set the coefficients of the rule so that this accommodates such expectations, the latter become self-fulfilling. Then the rule is unable to uniquely pin down the behavior of one or more real and/or nominal variables, making many different paths compatible with equilibrium (see Chari *et al.* (1998); CGG (2000); Carlstrom and Fuerst (2000); Svensson and Woodford (1999); and Woodford (2000)).

In order to gain insight into the stabilizing properties of IFB rules, following Batini and Pearlman (2002) we analyze their performance by using *root locus analysis*, a method that we borrow from the control engineering literature. Appendix A outlines how this method works. Use of this method allows us to identify analytically, for the most part, the range of stabilizing parameters (j, θ) in our sticky-price/sticky-inflation models for small values of parameters h and γ (the habit formation and price indexing parameters respectively) before indeterminacy sets in. It also proves to be a powerful method for computing threshold values for the general model. The method produces geometrical representations that show how system eigenvalues change as a function of the change in any parameter in the system. The technique involves starting from a polynomial equation and using a set of topological theorems to track the equation's roots as this parameter in the system varies. The locus describing the evolution of the roots when parameters change is called the 'root locus'. In our particular case we are interested in detecting how the characteristic roots of the model economy evolve as we vary the inflation feedback parameter θ, for given forecast horizons j in the policy rule. As the conditions for stability and determinacy of the model hinge on the value of these roots, from these diagrams we can infer which regions of the (j, θ) parameter space are associated with unique and well-behaved rational expectations equilibria. Since we condition on increasingly distant forecast horizons in the policy rule, the method entails deriving a separate diagram for each value of j. However, in the majority of cases a clear pattern emerges quickly, so in what follows we only draw these diagrams at most for $j = 0, 1,...,4$.

In the following subsections, we use the Aoki method to analyze separately the sum

and difference systems of two symmetric blocs pursuing symmetric IFB rules of the form (41) or (42). For open economies both sum and difference systems must be saddle-path stable for a stable and unique equilibrium. From the sum system (32) and (33), the central banks' choice of responding to consumer or price inflation as well as the existence of a home bias in consumption patterns are both irrelevant in the case of the sum system. In the case of the difference system this is no longer true, and so we investigate changes to these assumptions separately for that case.

4.2 The Sum System

The sum form of the IFB rule is given by

$$i_t^S = \rho i_{t-1}^S + \theta(1-\rho)\mathcal{E}_t \pi_{t+j}^S \tag{45}$$

Let z be the forward operator. Taking z-transforms of (32), (33) and (45), the characteristic equation for the sum system is given by:

$$\begin{aligned} & (z-\rho)[(z-1)(z-h)(\beta z-1)(z-\gamma) - \frac{\lambda}{\mu} z^2(\phi z + \mu(z-h))] \\ + \quad & \frac{\lambda\theta}{\mu}(1-\rho)(\phi z + \mu(z-h))z^{j+2} = 0 \end{aligned} \tag{46}$$

where we have defined $\lambda \equiv \frac{(1-\beta\xi)(1-\xi)}{\xi}$ and $\mu \equiv \frac{\sigma}{1-h}$. Equation (46) shows that the minimal state-space form of the sum system has dimension $\max(5, j+3)$. Since there are 3 predetermined variables in the sum system, it follows that the saddle-path condition for a unique stable rational expectations solution is that the number of roots inside the unit circle of the complex plane is 3 and the number of outside the unit circle is $\max(2, j)$.

To identify values of (j, θ) that involve exactly three roots of equation (46) we graph the root locus of (θ, z) pairs that traces how the roots change as θ varies between 0 and ∞. All the graphs can be drawn by following the rules set out in Appendix A. Other parameters in the system, including the feedback horizon parameter j in the IFB rule, are kept constant. We generate separate charts, each conditioning on a different horizon assumption. Each chart shows the complex plane (indicated by the solid thin line),[15] the

[15] In this plane, the horizontal axis depicts real numbers, and the vertical axis depicts imaginary numbers. If a root is complex, i.e. $z = x + iy$, then its complex conjugate $x - iy$ is also a root. Thus the root locus is symmetric about the real axis.

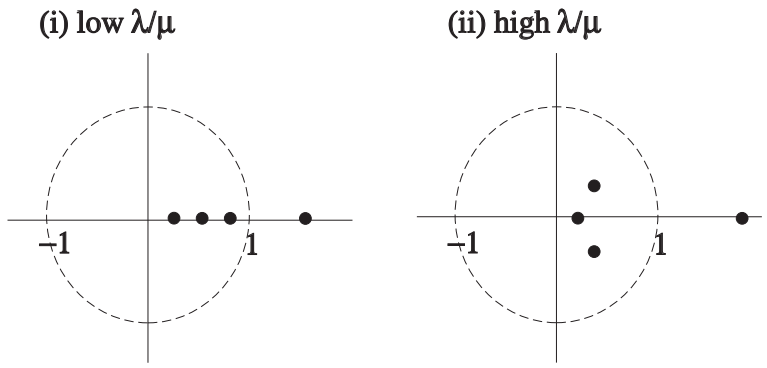

Figure 1: **Possible position of zeroes when** $\theta = 0$

unit circle (indicated by the dashed line), and the root locus tracking zeroes of equation (46) as θ varies between 0 and ∞ (indicated by the solid bold line). The arrows indicate the direction of the arms of the root locus as θ increases. Throughout we experiment with both a 'high' and a 'low' $\frac{\lambda}{\mu}$, as defined after (46). The economic interpretation of these cases is that the high $\frac{\lambda}{\mu}$ case corresponds to low ξ (i.e., more flexible prices) and low $\frac{\sigma}{1-h}$. From section 3.4 we have seen that the latter implies small spillover effects and hence low interdependence between the two blocs. Hence in the high $\frac{\lambda}{\mu}$ case, prices are relatively flexible and interdependence is relatively weak.

The term inside the square brackets in equation (46) corresponds to no nominal interest rate feedback rule (i.e., an open-loop interest rate policy). Then rule (41) or (42) is switched off and so the lagged term i^S_{t-1} disappears from our model; the system now requires exactly *two* stable roots for determinacy. Figure 1 plots the root locus in this case. Since with no policy θ is set to 0, the root locus is just a set of dots: namely, the roots of equation (46) when $\theta = 0$. Note that depending on the value of λ/μ, the position of these roots varies, and in the flexible price, low interdependence case where $\frac{\lambda}{\mu}$ is high, there are complex roots indicating oscillatory dynamics.[16] The diagram shows that there are *too many* stable roots in both cases (i.e. 3 instead of 2), which implies that with no interest rate feedback rule, there will always be indeterminacy in the sum system.

If the nominal interest rate rule is switched on and now feeds back on current rather than expected inflation, i.e. $j = 0$, then the root locus technique yields a pattern of zeros as depicted in Figure 2. Interest rate smoothing brings about a lag in the short-term

[16]How we find the position of these zeros is the main example of Appendix A.

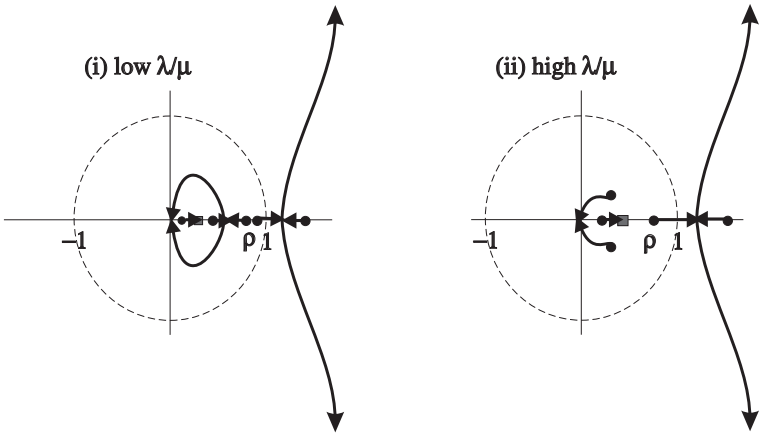

Figure 2: **Position of zeroes as θ changes using current inflation**

nominal interest rate and the system is now stable if it has exactly *three* stable roots (as we now have three predetermined variables in the system). The figure demonstrates that if θ is sufficiently large, one arm of the root locus starting originally at ρ exits the unit circle, turning one root from stable to unstable so that there are now three – as required – instead of four stable roots and the system has a determinate equilibrium. As $\theta \to \infty$, there are roots at $\pm i\infty$, two roots at 0, and one at $\mu h/(\phi + \mu)$, the latter shown as a square.

Note that when $\theta = z = 1$, the characteristic equation has the value 0, confirming that the branch of the root locus moving away from $z = \rho$ crosses the unit circle at a value $\theta = 1$. Thus we conclude that for a rule feeding back on current inflation, the sum system exhibits determinacy if and only if $\theta > 1$. For higher values of $j \geq 1$ we can draw the sequence of root locus diagrams shown in Figures 3-6, and so confirm the well-known 'Taylor Principle' that interest rates need to react to inflation with a feedback greater than unity. However for $j \geq 1$ our diagrams show that an arm of the root locus re-enters the unit circle for some high $\theta > 1$ and indeterminacy re-emerges. Therefore $\theta > 1$ is necessary but not sufficient for stability and determinacy. Our results up to this point are summarized in proposition 1:

***Proposition 1*: In the sum system, for a rule feeding back on current inflation ($j = 0$), $\theta > 1$ is a necessary and sufficient condition for stability and determinacy. For higher feedback horizons ($j \geq 1$), $\theta > 1$ is a necessary but not sufficient condition for stability and determinacy.**

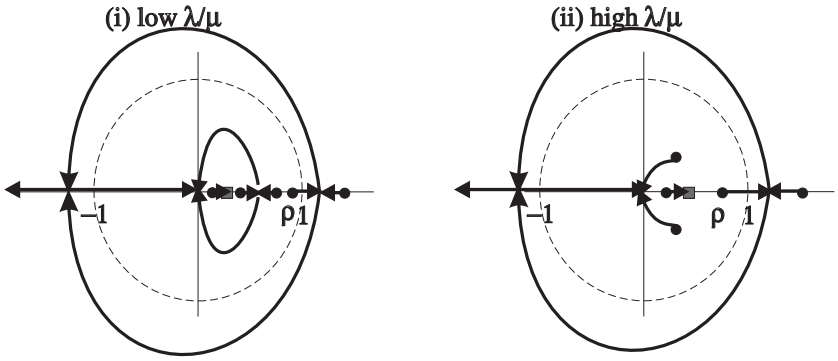

Figure 3: **Position of zeroes as θ changes: 1-period ahead expected inflation**

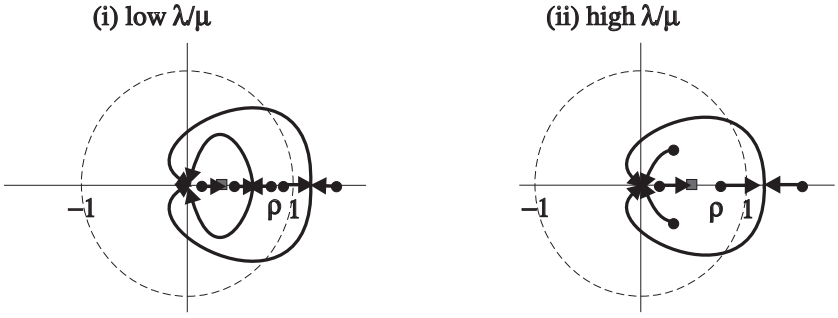

Figure 4: **Position of zeroes as θ changes: 2-period ahead expected inflation**

Now let $\theta^S(j)$ be the upper critical value of θ for the sum system for a feedback horizon j. Figure 3 shows that for the case $j = 1$, i.e. one-quarter ahead forecasts which corresponds to a case studied by CGG (2000), indeterminacy occurs when this portion of the root locus enters the unit circle at $z = -1$.[17] The critical upper value for $\theta = \theta^S(1)$ when this occurs is obtained by substituting $z = -1$ and $j = 1$ into the characteristic equation (46) to obtain:

$$\theta^S(1) = \frac{1+\rho}{1-\rho}\left[1 + \frac{2(1+h)(1+\beta)(1+\gamma)\mu}{\lambda(\phi+\mu(1+h))}\right] \tag{47}$$

[17] Thus Figure 3 portrays diagrammatically the result shown analytically by Woodford (2003), chapter 4, that there is a value of $\theta = \theta^S$ say, beyond which there is indeterminacy.

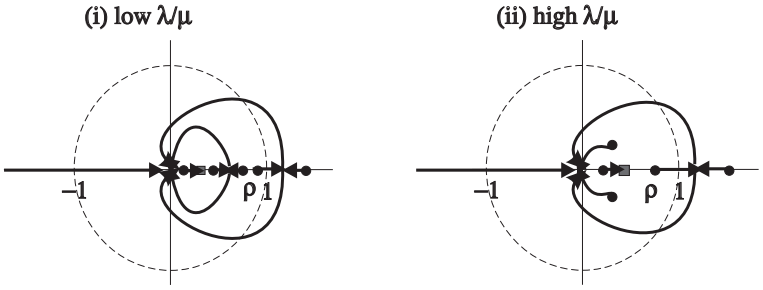

Figure 5: **Position of zeroes as θ changes: 3-period ahead expected inflation**

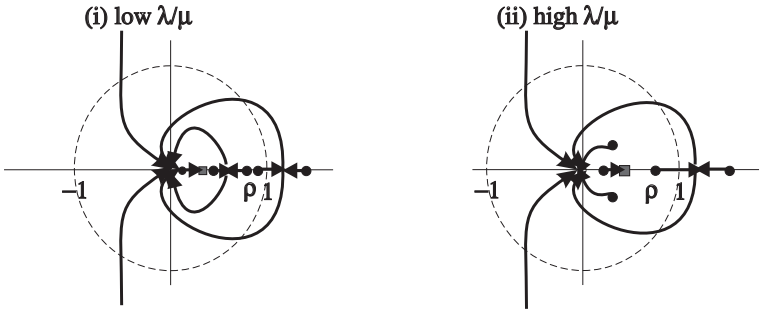

Figure 6: **Position of zeroes as θ changes: 4-period ahead expected inflation**

One important thing to note looking at this expression is that the greater is the degree of smoothing captured by the parameter ρ in the interest rate rule, the larger the maximum permissible value of θ before indeterminacy sets in. For $j \geq 2$, Figures 4-6 show that indeterminacy occurs when the root locus enters the unit circle at $z = cos(\psi) + isin(\psi)$ for some $\psi \in (0, \frac{\pi}{2})$. All our results up to this point are analytical using topological reasoning, but now the threshold $\theta^S(j)$ for $j \geq 2$ must be found numerically. Given j, write the characteristic equation as

$$\sum_{k=1}^{max(5,j+3)} a_k(\theta) z^k = 0 \tag{48}$$

noting that some of the a_k are dependent on θ. The root locus meets the unit circle at $z = cos(\psi) + isin(\psi)$. Using De Moivre's theorem $z^k = cos(k\psi) + isin(k\psi)$ and equating real and imaginary parts we arrive at two equations which can be solved numerically for θ and ψ. Results using MATLAB are reported in the next section.

As well as locating an upper threshold $\theta = \theta^S(j)$, an even more significant result concerning indeterminacy emerges from Figures 4, 5 and 6 for $j \geq 2$. These have been drawn in for values of ρ such that the two rightmost poles of the root locus are joined by

straight lines that meet *outside* the unit circle. The implication is that for some values of $\theta > 1$, these yield unstable roots of the system, and therefore the system will have exactly three stable roots which is what is required for determinacy. (Note that if the arms of the root locus from ∞ cross the unit circle before these latter meet, then there may anyway be too many stable roots). However, for a lower value of ρ it could happen that rather than meeting to the right of $z = 1$, the two arms instead meet to the left of $z = 1$, that is *inside* the unit circle and then remain within it, as in figure 7. This would imply that for all θ there are always more than three stable roots, which would entail, in turn, indeterminacy for all values of θ. We therefore conclude that there is determinacy for θ slightly greater than 1 if the root locus passes through $z = 1$ from the left, as in figures 3-6. Conversely, Figure 7 for the left and middle examples show indeterminacy for all θ if the root locus passes through $z = 1$ from the right. However, to be certain that this result is true for all θ, we need to be able to show that once this arm of the root locus enters the unit circle it never leaves it, as is not the case in the right hand example of Figure 7. The simplest case for which this 'pathological' behaviour cannot happen is when $h = \gamma = 0$. We can now show:

***Proposition 2*: For the general model there is always some lead J^S such that for**

$$j > J^S = \frac{1}{1-\rho} + \frac{(1-\beta)(1-\gamma)\sigma}{\lambda(\phi+\sigma)} \tag{49}$$

there is indeterminacy for all values of θ, provided that that the arm of the root locus from the right is 'non-pathological' in the sense that it enters the unit circle only once. If $h = \gamma = 0$ this is true if $\beta > \rho > \sqrt{2} - 1$ and $\frac{\lambda(\phi+\sigma)}{\sigma} > \frac{(1-\beta)(1+\rho)(1-\rho)^2}{\rho^2+2\rho-1}$.

Proof: See Appendix B.

For $h, \gamma > 0$ the derivation of sufficient conditions that rule out pathological behaviour has proved elusive. However for small values of h, γ, the root locus diagrams correspond to the 'low λ/μ' ones of Figures 2-6, with the inner arms that lie off the real axis becoming vanishingly small as h, γ tend to 0. By a continuity argument therefore, it follows that the sufficient conditions of Proposition 2 apply in this case as well for small h, γ. Numerical experiments indicate that pathological behaviour does not occur for all realistic values of

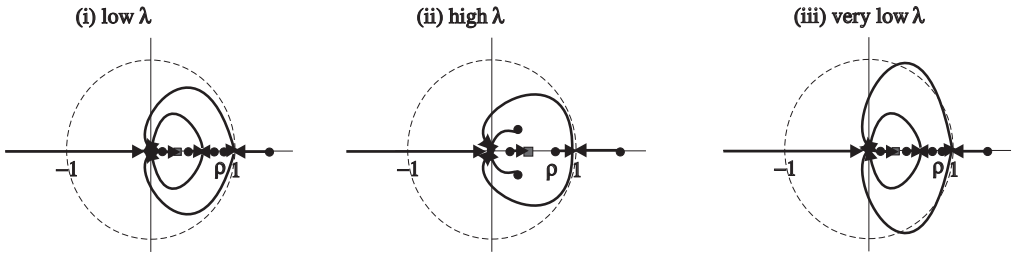

Figure 7: **Position of zeros as θ changes: 3-period ahead expected inflation, and low ρ**

the parameters.[18]Indeed it is extremely difficult to numerically produce diagrams such as that on the right-hand-side of figure 7. For example with other parameters set at central values the parameter ξ must exceed 0.9, corresponding the price contracts of 10 quarters. In addition our calibrated values indicate that the sufficient conditions in proposition 2 are easily satisfied.

Propositions 1 and 2 confirm, in a rigorous setting, the possibility of real indeterminacy for any IFB rule with lead $j \geq 1$ when the feedback on producer price inflation is below unity (the Taylor principle) and above a threshold $\theta^S(j)$. The root locus diagrams in figures 3 and 4 show that $\theta^S(1) > \theta^S(2)$, so that indeterminacy becomes more of a problem as j increases from $j = 1$ to $j = 2$. Table 1 below shows that this deterioration continues for higher j and eventually, from proposition 2, for high j no IFB rule of the form (42) results in a unique stable equilibrium. The value of ρ is crucial in determining the critical value of the lead j beyond which indeterminacy sets in. The lower ρ, the lower the maximum-permitted inflation horizon the central bank can respond to, and hence, the larger the region of indeterminacy under IFB rules. Thus the absence of interest rate smoothing has the same indeterminacy-inducing effect as high j.

[18]BLP provides calibrated values for parameters mostly based on Smets and Wouters (2003). For the US central values for parameters are $\rho = 0.9$, $\beta = 0.99$, $\sigma = 2$, $\phi = 0.8$, $\gamma = 0.5$, $\xi = 0.75$ and $h = 0.6$, assuming a quarterly model. Then $\lambda = 0.086$ and $\mu = 5$.

4.3 The Difference System

In this section we analyze the effect of the IFB rule in the difference system. We shall see that, in this case, there are important differences in the conditions for determinacy depending on (i) whether the central banks react to producer or consumer price inflation and on (ii) the degree of openness of the two economies (as captured by the parameter ω). We start by considering the case of complete integration (i.e. $\omega = \frac{1}{2}$ and no home bias), looking first at IFB rules based on producer price inflation and then at IFB rules based on consumer price inflation. Then we consider the case when there is home bias, however restricting ourselves to the case of no habit formation ($h = 0$) and a unit elasticity of substitution in the utility function ($\sigma = 1$). These more restrictive assumptions imply no foreign asset dynamics about a balanced trade steady state (since trade is always balanced), as when we assumed no home bias. Without these restrictions we need to address the well-known problems associated with Ramsey consumers in open economies (see, for example, Schmitt-Grohe and Uribe, 2001).[19]

4.3.1 No Home Bias and IFB Rules Based on Producer Price Inflation

With interest rates feeding back on producer price inflation, the IFB rule in difference form is given by

$$i_t^D = \rho i_{t-1}^D + \theta(1-\rho)\mathcal{E}_t \pi_{t+j}^D \tag{50}$$

Taking z-transforms of (50), (39) and (40), it is now easy to show that for the difference system the characteristic equation reduces to

$$(z-\rho)[(z-1)(\beta z-1)(z-\gamma) - \lambda(1+\phi)z^2] + \lambda\theta(1-\rho)(1+\phi)z^{j+2} = 0 \tag{51}$$

The root locus diagrams for this characteristic equation will have qualitatively the same features as those for the sum system. So propositions 1 and 2 apply to the difference system as well. Again numerical results rule out pathological behaviour of the root loci. By analogy with our earlier results, the critical upper value $\theta^D(1)$ for the difference system

[19] An alternative way of handling the foreign assets problem is to follow BB and CGG, among others, and recast the model as stochastic with complete asset markets. The linearized stochastic model has an identical deterministic component and therefore the stability analysis, which is all that concerns us in this paper, all goes through as before. Furthermore, in that case the analysis is possible without restrictions on h and σ for the home bias case.

when both central banks respond to producer price inflation with a feedback horizon $j = 1$ is given by

$$\theta^D(1) = \frac{1+\rho}{1-\rho}\left[1 + \frac{2(1+\beta)(1+\gamma)}{\lambda(\phi+1))}\right] \tag{52}$$

and a sufficient condition for indeterminacy is now:

$$j > J^D = \frac{1}{1-\rho} + \frac{(1-\beta)(1-\gamma)}{\lambda(1+\phi)} \tag{53}$$

It follows from a little algebra that $\theta^S(1) > \theta^D(1)$ iff $\sigma > \frac{1-h}{1+h}$ and that $J^S > J^D$ iff $\sigma > 1$. In our calibration in Appendix C we report estimates for σ well above unity. So for $h \approx 0.5$, we conclude that $\theta^S(1) > \theta^D(1)$ and $J^S > J^D$ for plausible parameter values. For $j \geq 2$, threshold values must be computed numerically. Figure 8 shows the areas of stability and determinacy in (j, θ) space for the sum and difference systems. The figure indicates that the area of indeterminacy is smaller for the difference system case. In our open economy model, both the sum and difference systems must be stable and determinate for the world economy to have this property. Our results indicate that in this respect the constraints on (j, θ) for the difference system are the binding ones. Furthermore our expressions for $\theta^S(1)$, $\theta^D(1)$, J^D and J^S indicate that as σ and h increase, the parameter space associated with determinate equilibria under an IFB rule shrinks in the open-economy relative to the closed-economy case. We synthesize these results via the following proposition:

Proposition 3. **With IFB rules responding to producer price inflation and with no home bias, if $\sigma > 1$ then potential indeterminacy is exacerbated in the open economy, and it becomes worse as σ and the habit parameter h increase.**

To see the intuition behind this result one needs to go back to the spill-over effects of monetary policy captured by the parameter κ_1 defined in (31). There we saw that as $\frac{\sigma}{1-h}$ increases, then the negative spillover effects dominate and the stabilizing effect of the IFB rule in one bloc has the opposite effect in the other bloc. Thus the rule has a beggar-thy-neighbour character leading to possibly incompatible responses to shocks and the absence of a unique stable equilibrium, i.e., indeterminacy. Figure 8 illustrates proposition 3 by showing $\theta^S(j)$ and $\theta^D(j)$. As the proposition suggests, the area of indeterminacy is larger in the open-economy case (this area now being equivalent to the sum of the dark and

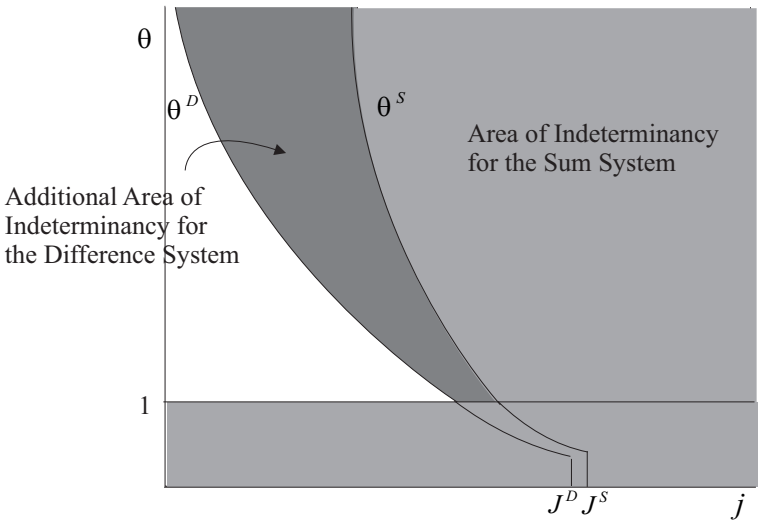

Figure 8: **Areas of Determinacy for the Sum Difference Systems: Feedback on Producer Price Inflation and No Home Bias.**

light grey areas in the diagram) than in the closed-economy case. As σ and h grow in magnitude, the dark area in the diagram expands, thus increasing the negative output spillovers between the two blocs. Also from (49) and (53) as interest rate smoothing ρ increases, both $\theta^S(j)$ and $\theta^D(j)$ shift to the right alleviating the indeterminacy problem for both closed and open economies alike. Table 1 quantifies numerically upper critical values for θ in the sum and difference system cases, respectively when we calibrate the model's parameters as described in Appendix C of BLP using US data (see footnote 18).

j	j=1	j=2	j=3	j=4	j=5	j=6	j=7	j=8	j=9	j=10	j=11
$\theta^S(j)$	369	60.2	12	5.5	3.5	2.62	2.05	1.67	1.40	1.18	1.02
$\theta^D(j)$	247	38.2	9.6	5.1	3.4	2.57	2.04	1.66	1.39	1.18	1.02

Table 1. Critical upper bounds for $\theta^S(j)$ and $\theta^D(j)$.

4.3.2 No Home Bias and IFB Rules Based on Consumer Price Inflation

With no home bias purchasing power parity applies to the consumer index and therefore $\pi_t - \pi_t^* = \Delta s_t$. Hence using (38) the interest rate rule of the difference system is given by

$$i_t^D = \rho i_{t-1}^D + \theta(1-\rho)\mathcal{E}_t \Delta s_{t+j} = \rho i_{t-1}^D + \theta(1-\rho)\mathcal{E}_t i_{t+j-1}^D \tag{54}$$

where $i_t^D \equiv i_t - i_t^*$. With the nominal interest rate (in difference system form) depending only on leads and a lag of itself, the policy reaction function is completely decoupled from the rest of the difference system. This leads to the result:.[20]

***Proposition 4*: When IFB rules in the two blocs respond to consumer price inflation and there is no home bias in consumption, a rule for both blocs feeding off consumer price inflation expected at *any* time horizon $j \geq 0$ leads to indeterminacy of the equilibrium.**

Proof: From (54), i^D is completely decoupled from y^D and π^D. It therefore follows that the joint determinacy properties of (37) and (40) are completely independent of i^D, because we can treat the latter as an exogenous variable. The relevant characteristic equation is then given by

$$(\beta z - 1)(z - 1)(z - \gamma) - \lambda(1 + \phi)z^2 = 0 \tag{55}$$

Root locus analysis of this equation for values of λ ranging from 0 to ∞ show that there are always two stable roots, whereas inspection of (39) and (40) shows that determinacy requires one stable root. Hence the system is always indeterminate.[21] $\square$

The intuition behind this results follows from that for IFB rules feeding back on producer price inflation, as in proposition 3. Now, since targeting consumer price inflation in effect adds an nominal exchange rate target, the beggar-thy-neighbour character of the rules is exacerbated. On bloc's appreciation to reduce consumer price inflation has the opposite effect on the second bloc and the conflict between the responses of the two monetary authorities is now incompatible with any saddle-path stable equilibrium. However, as we show in our final subsection, this extreme result is a consequence of the complete openness of the two economies.

[20] See Zanna (2003) for a discussion of how conditions for determinacy are affected by the choice of the inflation measure to which the central bank responds to in the small-open economy case.

[21] Note that the decoupled interest rate process has a characteristic equation $z - \rho - \theta(1 - \rho)z^j = 0$. By the root locus method it can be shown that this system also has an indeterminate equilibrium for $j > 1$ and for $j = 1$ when $\theta > \frac{1-\rho}{1+\rho}$. However, for the system as a whole the indeterminacy is determined by that of the y^D, π^D system as given in the proof.

4.3.3 The Effect of Home Bias

As discussed earlier, allowing for home bias in consumption patterns has no implications for the sum system, and we therefore only need to consider its impact on the difference system. In this system, we can ignore problems arising from foreign asset dynamics by focussing on the case $\sigma = 1$ and $h = 0$. Writing $\tau_t = p_{Ht} - s_t - p^*_{Ft}$ in linearized form, this yields a representation for the difference system:

$$\begin{aligned}
(2\omega - 1)\tau_t &= c_t^D && (56)\\
y_t^D &= (1-2\omega)c_t^D - 4\omega(1-\omega)\tau_t = -\tau_t && (57)\\
(1+\beta\gamma)\pi_t^D &= \beta E_t\pi_{t+1}^D + \gamma\pi_{t-1}^D + \lambda(-(1+\phi)a_t^D + c_t^D + \phi y_t^D - 2\omega\tau_t) \\
&= \beta E_t\pi_{t+1}^D + \gamma\pi_{t-1}^D - \lambda(1+\phi)(\tau_t + a_t^D) && (58)
\end{aligned}$$

Consider first feedback from forward-looking producer price inflation, given for the difference system by (50). Together with (56) and the UIP condition, which we write in terms of the terms of trade as

$$\mathcal{E}_t(\tau_{t+1}) - \tau_t = \mathcal{E}_t\pi_{t+1}^D - i_t^D \tag{59}$$

this generates a characteristic equation identical to that for no home bias, (51). Thus with $h = 0$ and $\sigma = 1$ and feedback from producer price inflation, the conditions for indeterminacy are not affected by the existence of home bias.

For the case of feedback from forward-looking consumer price inflation, we can use (59) to write the difference system for interest rates as

$$i_t^D = \rho i_{t-1}^D + \theta(1-\rho)(2\omega\mathcal{E}_t i_{t+j-1}^D + (1-2\omega)\mathcal{E}_t\pi_{t+j}^D) \tag{60}$$

This leads to a characteristic equation given by

$$(z-\rho)[(\beta z-1)(z-1)(z-\gamma)-\lambda(1+\phi)z^2]-\theta(1-\rho)z^j[2\omega(\beta z-1)(z-1)(z-\gamma)-\lambda(1+\phi)z^2] = 0 \tag{61}$$

Inspection of the system of dynamic equations (58), (59) and (60), shows that determinacy requires exactly two stable roots. For the case $j = 1$, the root locus diagram Figure 9 shows that this is the case for a large range of $\theta > 1$. Note that there is a branch point into

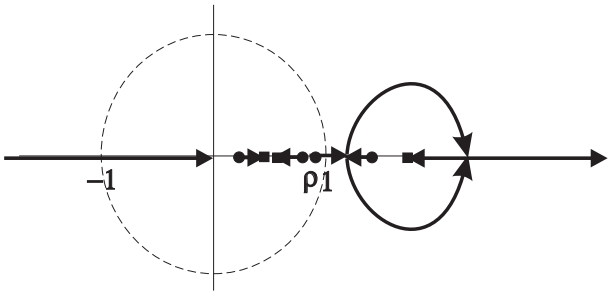

Figure 9: **Position of zeroes as θ changes, for $j = 1$ in the home bias difference system with CPI inflation based IFB rules.**

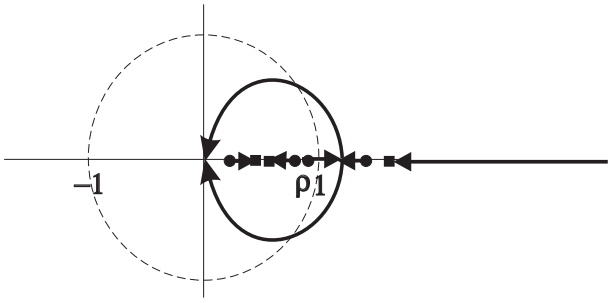

Figure 10: **Position of zeroes as θ changes, for $j = 2$ in the home bias difference system with CPI inflation based IFB rules.**

the complex plane, which returns to the real axis for a larger value of θ; as θ approaches a further critical value, one of the zeroes tends to ∞, and beyond this critical value it heads along the real axis from $-\infty$. Finally, there is a critical value of θ at which $z = -1$, and any higher values of θ yield indeterminacy. For $j = 1$ we can evaluate the upper bound on θ as before by putting $z = -1$ and $j = 1$ in (61). For the case under consideration with feedback from consumer price inflation and home bias $\omega \neq \frac{1}{2}$, denote this threshold at $j = 1$ by $\theta^D(CP, \omega)$. Then we obtain:

$$\theta^D(CP, \omega) = \frac{1+\rho}{1-\rho}\left[1 + \frac{2(1-2\omega)(1+\beta)(1+\gamma)}{4\omega(\beta+1)(1+\gamma) + \lambda(\phi+1))}\right] \tag{62}$$

For $j = 2$, from Figure 10 the critical value at which indeterminacy occurs is not associated with $z = -1$. Similar root locus diagrams to the ones we have seen earlier can then be drawn for values of $j > 2$ we can now show that indeterminacy occurs for all $\theta > 1$, provided that the derivative of the LHS of (61) at $\theta = z = 1$ is greater than 0. The

threshold values of j must then satisfy

$$j > \frac{1}{1-\rho} + \frac{(1-\beta)(1-2\omega)(1-\gamma)}{\lambda(1+\phi)} = J^D(CP,\omega) \tag{63}$$

where we denote the threshold horizon for the case of feedback from consumer price inflation with home bias by $J^D(CP,\omega)$. Note that these results do not apply when there is no home consumption bias ($\omega = 1/2$), because this is a knife-edge case in which nominal relative interest rates are decoupled from the rest of the system.

We can now compare the difference systems with home bias under rules based on producer price, and on consumer price inflation. Denote the θ-threshold at $j = 1$ and the j-threshold for producer price based rules by $\theta^D(PP,\omega)$ and $J^D(PP,\omega)$ respectively. We have shown that for $h = 0$ and $\sigma = 1$ we obtain $\theta^D(PP,\omega) = \theta^D$ and $J^D(PP,\omega) = J^D$ obtained previously without home bias. Gathering together these results, after some algebra we arrive at:

$$\theta^D(PP,\omega) - \theta^D(CP,\omega) = \frac{4(1+\rho)(1+\beta)(1+\gamma)\omega[2(1+\beta)(1+\gamma)+\lambda(1+\phi)]}{(1-\rho)\lambda(1+\phi)[4\omega(1+\beta)(1+\gamma)+\lambda(1+\phi)]} \tag{64}$$

$$J^D(PP,\omega) - J^D(CP,\omega) = \frac{2\omega(1-\beta)(1-\gamma)}{\lambda(1+\phi)} \tag{65}$$

Clearly $J^D(PP,\omega) - J^D(CP,\omega)$ increases with $\omega \in [0, \frac{1}{2}]$ as we proceed from autarky to a complete integration of the two economies. It is easy to show that the same is true for $\theta^D(PP,\omega) - \theta^D(CP,\omega)$. By analogy with the reasoning leading up to proposition 3, we have a proposition that qualifies proposition 4 by considering less that completely open economies:

Proposition 5. **Confining ourselves to the case $\sigma = 1$, $h = 0$, with home consumption bias, the potential indeterminacy of IFB rules is worse when based on consumer rather than producer price inflation, and becomes increasingly worse as the degree of openness of the two blocs increases.**

5 Conclusions

This paper has examined conditions for a unique stable rational expectations equilibrium for a symmetric two-bloc world economy where monetary authorities in both blocs pursue

IFB rules. Most of the literature in this area assumes that the economy is closed. In the open economy changes to nominal interest rate affect aggregate demand through both intertemporal substitution effects (as in a closed economy) and terms of trade effects, working in opposite directions. Given the additional terms of trade effect, it is reasonable to expect that IFB rules would perform differently in the open economy, and indeed we find this to be the case.

Our results are best synthesized by focussing on the critical upper bound for the expected inflation feedback parameter beyond which there is indeterminacy, $\theta^S(j)$ and $\theta^D(j)$ for the sum and difference systems respectively, where j is the feedback horizon. The diverse performance of rules in the closed and open economy can be summarized by the difference $\theta^S(j) - \theta^D(j)$. Consider first the case when there is no home bias and the degree of openness is at its maximum. For IFB rules based on producer price inflation this difference is positive, indicating that indeterminacy is a more serious problem for the open economy. If rules are based on consumer price inflation the problem worsens; indeed, in the case of no home bias, an IFB rule responding to consumer price inflation at *any* horizon $j \geq 0$ (i.e., including feedback on current consumer price inflation) leads to indeterminacy.[22] With consumer price inflation feedback and some home bias, the indeterminacy problem is less severe, but it rapidly deteriorates towards the extreme case as the bias diminishes and the economies become more open, since in that case the $\theta^S(j) - \theta^D(j)$ increases. The rationale behind the poorer performance of IFB rules based on consumer price inflation lies with beggar-thy-neighbour behavior. between two blocs when central banks simultaneously attempt to lower domestic consumer price inflation, now including an imported component, by improving their own bloc's terms of trade.

Although the euro area and the US are not very open, and so they probably do not fall foul of our worst case scenario, our results are nevertheless an important warning for the ECB and the Federal Reserve, since they imply that concurrent excessive preemptiveness in response to shocks may expose both to self-fulfilling sunspot sequences for any feedback on inflation forecasts. Since both the ECB and the Federal Reserve focus primarily on consumer price inflation [23] and not on producer price inflation, our results on the poor

[22] In fact it is straightforward to show that proposition 4 also holds for any backward lag, $j < 0$.

[23] As measured respectively by changes in the Harmonized Index of Consumer Prices, HICP; and changes in the Personal Consumption Expenditure, PCE, in the form of either the chain-weighted index or the

performance of consumer price based rules also have normative implications.

A A Topological Guide to The Root Locus Technique

Here we present a brief guide to how to use the root locus technique. We start by some standard 'rules' as provided in control theory textbooks, and then apply them to a specific example.

The idea is to track the roots of the polynomial equation $f(z) + \theta g(z) = 0$ as θ moves from 0 to ∞. Clearly for $\theta = 0$, the roots are those of $f(z) = 0$, whereas when $\theta \to \infty$, the roots are those of $g(z) = 0$. The root locus then connects the first set of roots to the second set by a series of lines and curves. We shall assume without loss of generality that the coefficient of the highest power of each of f and g is unity.

There are a number of different ways of stating the standard control 'rules' that underly the technique. One popular way (see Evans (1954)) involves just 7 steps:

1(a). Define $n(f) =$ no. of zeros of $f(z)$, $n(g) =$ no. of zeros of $g(z)$.

1(b). Loci start at the zeros of $f(z)$, and end at the zeros of $g(z)$ and at ∞ if $n(f) > n(g)$.

1(c). Loci start at the zeros of $f(z)$ and at ∞, and end at the zeros of $g(z)$ if $n(g) > n(f)$.

2. Number of loci must be equal to $\max(n(f), n(g))$.

3. A point on the real axis is on the root locus if the number of zeros of f and g on the real axis to its right is odd.

4. Loci ending or beginning at ∞ do so at angles to the $+ve$ real axis given by $(2k+1)\pi/(n(p) - n(z))$, where k goes from 0 to $(n(p) - n(z))$.

5. Asymptotes at ∞ intersect the real axis at the center of gravity of the zeros of f and g, i.e. [Sum of zeros of f - Sum of zeros of g]$/(n(f) - n(g))$.

6. If all coefficients of f and g are real, then the root locus is symmetric about the real axis.

7. Loci leave the real axis where $\partial\theta/\partial z = 0$.

A specific example is provided by (46) without an interest rate rule:

$$(z-1)(z-h)(\beta z - 1)(z - \gamma) - \frac{\lambda}{\mu} z^2(\phi z + \mu(z-h)) = 0$$

Consider changes to $\frac{\lambda}{\mu}$. Then $f(z)$ in the notation above has roots at 1, at h, γ both inside the unit circle, and at $\frac{1}{\beta}$ outside the unit circle, while $g(z)$ has two roots at 0 and one at $\frac{\mu h}{\phi + \mu}$, which is less than h. Thus by Step 1(b), there will be a root at ∞. The root locus diagrams in the main text have been drawn for the case $\gamma < \frac{\mu h}{\phi + \mu}$, so we assume this for the moment.

deflator.

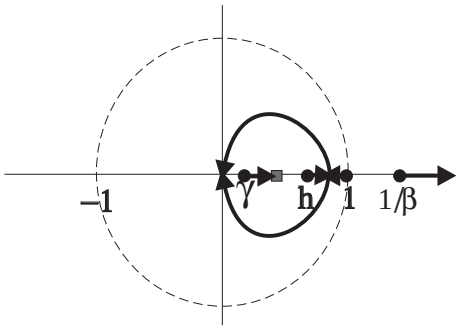

Figure 11: Position of zeros for varying $\frac{\lambda}{\mu}$

By examination of the characteristic equation we see that as $\frac{\lambda}{\mu} \to \infty$, there is a root at $+\infty$; there is no logical possibility that it could be connected to any of the roots other than $\frac{1}{\beta}$, otherwise Step 6 would be violated. Secondly we note that there cannot be an arm of the root locus connecting γ to 0, because it would then be impossible for either arm starting at 1 or at h to also get to 0, without again violating Step 6. It therefore follows that there must be an arm connecting γ to $\frac{\mu h}{\phi+\mu}$. In order for the arms starting at 1 and h to then get to 0, they must head towards one another and then branch off into the complex plane. Logically therefore, there is only one way of drawing the diagram, as shown.

This diagram explains the position of the zeros as depicted in Figure 11 for low and high $\frac{\lambda}{\mu}$. Note that if $\gamma > \frac{\mu h}{\phi+\mu}$, it is easy to show that the root locus diagram changes very little. γ will still have an arm connecting it to $\frac{\mu h}{\phi+\mu}$, but the arrow will point in the opposite direction.

B Proof of Proposition 2

We prove this in several stages. Firstly we find the conditions that ensure that the root locus crosses the unit circle from the right. Then we derive the sufficient conditions that ensure that this arm of the root locus never leaves the unit circle.

Write (46) as $f + \theta g = 0$. Taking derivatives with respect to θ, and evaluating at $\theta = 1, z = 1$ yields $[f'(1) + g'(1)]\frac{\partial z}{\partial \theta} + g(1) = 0$. By inspection $g(1) > 0$, so that the root locus crosses $z = 1$ from the right if $f'(1) + g'(1) > 0$. Substituting from (46) and rearranging, this is a requirement that (49) is satisfied.

For the next stage of the proof we require the following two results:

***Lemma 1*: The arms of the root locus for j and $j + 1$ never intersect in the complex plane.**

Proof: Suppose that the root loci meet at a value z^* where the corresponding θ values for $j, j + 1$ are given by θ_j, θ_{j+1}. It then follows that $\theta_{j+1} z^* = \theta_j$, which implies that z^* must be real and not complex.

The corollary to this result is that the arms of the root locus that lie on the real line are common to all j.

***Lemma 2*: The arms of the root locus that branch out from the real axis and then head to 0 for a given j, enclose the the corresponding arms for $j+1$.**

Proof: From Lemma 1, all we need is to find one point on these arms for which this is true. Accordingly, we show that this is the case for the branch points into the complex plane, denoted z_j. Such a branch point occurs where the derivative of the characteristic equation is equal to 0. Denoting the characteristic equation by $\theta z^{j+1} + h(z) = 0$, the value of θ at the branch point, together with the value z_j, is obtained by solving

$$\theta_j z_j^{j+1} + h(z_j) = 0 \qquad (j+1)\theta_j z_j^j + h'(z_j) = 0 \tag{B.1}$$

To prove the result, we now need to show that z_{j+1} is to the left of z_j. This can be done by demonstrating that the root locus passes through z_j from the right, which is equivalent to $\frac{\partial z}{\partial \theta} < 0$ at z_j. This derivative is obtained by total differentiation of the characteristic equation for $j+1$:

$$[(j+2)\theta z^{j+1} + h'(z)]\frac{\partial z}{\partial \theta} + z^{j+2} = 0 \tag{B.2}$$

Since the branch point of interest is positive, all we need to show therefore is that $(j+2)\theta z^{j+1} + h'(z) > 0$ at $z = z_j$. But from the proof to Lemma 1, we know that $\theta z_j = \theta_j$, so that $(j+2)\theta z_j^{j+1} + h'(z_j) = (j+2)\theta_j z_j^j + h'(z) = \theta_j z_j^j > 0$, using (B.1).

Remainder of Proof of Proposition 2: The sequence of root locus diagrams corresponding to this special case $h = \gamma = 0$ of our model is very similar to those of Figures 2-7. The key differences are that the inner arms of the root locus that branch into the complex plane are absent, while the very short arm that lies along the real axis inside the unit circle now ends at $z = 0$. Furthermore, because the only dynamics in this situation arise from the interest rate rule, there is now only one predetermined variable for the system. We shall establish a sufficient condition for $\lambda(\phi+\sigma)/\sigma$, which for convenience we define as Λ. We first note that after setting $h = \gamma = 0$ and dividing by z^2, (46) can be rearranged as

$$Kz^{j+1} + (z-\rho)[(z-1)(\beta z - 1) - \Lambda z] = 0 \tag{B.3}$$

where K is appropriately defined. We are now interested in the points where the root locus crosses the unit circle; these are given by $z = e^{i\psi}$, where ψ is the angle measured from the real axis. Noting that $e^{in\psi} = \cos(n\psi) + i\sin(n\psi)$, we can solve for K and ψ simultaneously by writing (B.3) as two separate equations, one involving cos and the other sin terms. K can be eliminated by multiplying the sin equation by $\cos((j+1)\psi)$, and subtracting it from the cos equation multiplied by $\sin((j+1)\psi)$. This yields an equation of the form

$$\beta\sin((j-2)\psi) - (\beta+\beta\rho+1+\Lambda)\sin((j-1)\psi) + (\beta\rho+\rho+1+\rho\Lambda)\sin(j\psi) - \rho\sin((j+1)\psi) = 0 \tag{B.4}$$

All the solutions other than $\psi = 0, \pi$ can in principle be found by dividing this equation by $\sin\psi$, and expressing it as a polynomial equation in $\cos\psi$ of order j. By drawing two root locus diagrams in the manner shown in the main text, one for positive values of K and the other for negative values of K, it is straightforward to account for $j-2$ solutions of (B.4).

A sufficient condition for no further crossings of the unit circle is to have parameter values such that the last two of the solutions for (B.4) for $\cos\psi$ are greater than 1. Now denote the LHS of (B.4) after division by $\sin\psi$ by $f(\cos\psi)$. Noting that (a) the coefficient of the highest power of $\cos\psi$ in f is proportional to the coefficient on $\sin((j+1)\psi)$, and is therefore negative, so $f(x)$ tends to $-\infty$ as x tends to ∞, and (b) $f(1) = -(1-\rho)\Lambda < 0$, it follows that a sufficient condition for two roots of f being greater than 1 is that $f'(1) > 0$. We shall impose this sufficient condition on the critical value of j of the text, which has the property:

$$j > \frac{1}{1-\rho} + \frac{1-\beta}{\Lambda} > j-1 \tag{B.5}$$

To calculate $f'(1)$, we use the result that $\frac{d}{d(\cos\psi)}\frac{\sin(n\psi)}{\sin\psi}\Big|_{\cos\psi=1} = \frac{n^3-n}{3}$. Substituting into (B.4) we obtain

$$3f'(1) = -\Lambda(1-\rho)j^3 + 3j^2(\Lambda + (1-\beta)(1-\rho) + j(9\beta - 3\beta\rho - 3 - 3\rho - 2\Lambda - \rho\Lambda) - 6\beta \tag{B.6}$$

Now substitute $-j > -1 - \frac{1}{1-\rho} - \frac{1-\beta}{\Lambda}$ to turn the $-j^3$ term into a j^2 term; then substitute $j > \frac{1}{1-\rho} + \frac{1-\beta}{\Lambda}$ first to eliminate the j^2 term, and then to eliminate the j term, ignoring the terms with denominator Λ (all positive provided that $\beta > \rho$), which yields

$$3f'(1) > \frac{\Lambda(\rho^2 + 2\rho - 1)}{(1-\rho)^2} - (1-\beta)(1+\rho) \tag{B.7}$$

which is positive provided that the sufficient condition in proposition 2 is satisfied.

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